# Technical Notes

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## Mesolayer Theory for Turbulent Flows

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### I. Introduction

LONG and Chen,[1] Afzal,[2,3] and Afzal and Bush[4] have reported the intermediate layer theory for turbulent shear flows. The velocity in the intermediate (meso) layer, as postulated by Long and Chen,[1] is of the order of the friction velocity. The inner and intermediate layers in the mesolayer theory[1] were regarded as a composite expansion that was matched with the outer defect layer. Their theory showed that for a boundary layer the logarithmic region was absent and that for a pipe flow the effect of the mesolayer was weak, although it did tend to modify the classical logarithmic behavior. In contrast to Long and Chen, Afzal[2,3] has shown that the velocity in the intermediate layer is of the order of unity. Afzal's theory[2,3] dealt separately with three (inner, intermediate, and outer) layers that were matched in the two overlap domains to show the possibility of two logarithmic regions rather than the one proposed by classical theory. Afzal[2,3] and Afzal and Bush[4] have shown that, in terms of classical theory, there is a distinct intermediate limit between the outer defect layer and the inner wall layer. This (new) three-layer theory modifies and/or extends the (old) two-layer theory. The implications of turbulent modeling by eddy viscosity and mixing length have been studied by Afzal and Bush.[4]

The aim of this Note is to modify Long and Chen's mesolayer theory in the light of Afzal's proposition[2,3] that the velocity in the intermediate layer is of the order of $U_c$, where $\mathcal{O}(U_c) > \mathcal{O}(u_\tau)$. On the basis of the above stated proposition, a composite expansion for the inner and intermediate layer is formulated. The composite expansion is matched with the outer defect layer to show that the classical logarithmic region always exists, provided $U_c$ is judiciously selected. It is further shown that the intermediate layer, when analyzed in terms of the two-layer classical theory, plays an insignificant role as far as the lowest-order results are concerned. However, this intermediate layer plays a significant role in the first-order theory where a three-layer picture of the flow is required.

### II. Governing Equation

Fully developed turbulent pipe/channel flow is governed by

$$\rho\nu\frac{\mathrm{d}u}{\mathrm{d}y}+\tau=\tau_w\left(1-\frac{y}{h}\right) \tag{1}$$

$$y=0 \qquad u=\tau=0 \tag{2}$$

$$y=h \qquad u=U, \quad \tau=0 \tag{3}$$

where $u$ is the mean velocity in axial direction, $U$ the centerline velocity, $\tau$ the appropriate Reynolds stress, $y$ the normal coordinate measured from the wall, $h$ the pipe radius/semidepth of the channel, $\tau_w$ the wall shear, $\nu$ the kinematic viscosity, and $\rho$ the density. The appropriate Reynolds number $R_\tau$ is defined as

$$R_\tau = u_\tau h/\nu \tag{4}$$

where $u_\tau = (\tau_w/\rho)^{1/2}$ is the friction velocity.

### III. Three Layers

A brief description of the three-layer structure of turbulent pipe flow considered by Afzal[2] is described below. The length scales of inner, intermediate, and outer layers, respectively, are

$$\eta = YR_\tau, \qquad \zeta = YR_\tau^{1/2}; \qquad Y=y/h \tag{5}$$

#### Inner Layer

The inner limit is defined as $R_\tau \to \infty$ with $\eta$ fixed. In the inner layer, the molecular and eddy transports are of the same order. To the lowest order, the inner expansions are

$$u=u_\tau f(\eta) \tag{6a}$$

$$\tau=\tau_w(\eta) \tag{6b}$$

and total stress remains constant.

#### Intermediate Layer

The intermediate limit is defined as $\zeta$ fixed for $R_\tau \to \infty$. The intermediate expansions are

$$u=U_c\mathfrak{F}_0(\zeta)+u_\tau\mathfrak{F}_1(\zeta)+\ldots \tag{7a}$$

$$\tau=\tau_w[1+R_\tau^{-1/2}\mathcal{g}(\zeta)+\ldots] \tag{7b}$$

where $U_c$ is of the order of unity. In this layer, to the lowest-order the Reynolds stress gradient is dominant and to the next order viscous, pressure gradient and perturbed Reynolds stress gradient terms are of the same order.

#### Outer Layer

The outer limit is defined as $Y$ fixed for $R_\tau \to \infty$. In this layer the Reynolds stress and pressure gradients are of the same order and the eddy transport dominates the molecular transport. The outer expansions are

$$u=U+u_\tau F(Y) \tag{8a}$$

$$\tau=\tau_w G(Y) \tag{8b}$$

#### Results

The matching of the three layers is carried out in the two overlap domains. Under certain conditions the analysis leads

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to the classical skin friction law, the law of the wall in the inner layer, and the velocity defect law in the outer layer. In the intermediate layer, the velocity is governed by a half defect law and the Reynolds stress is associated with a maximum.

## IV. Analysis

In this section the mesolayer theory[1] is modified by considering appropriate composite expansion for inner and intermediate layers, leading to the results described in Sec. III. Any series that reduces to the limiting asymptotic expansions when expanded asymptotically for $R_\tau \to \infty$ in corresponding limiting variable is known as a composite expansion.[5,6] In a composite expansion, each term is composed of an inner part and an outer part (Ref. 5, p. 13) and can a priori be conceived if the asymptotic structure of the associated layers is known. The composite expansion for velocity, in the inner and intermediate layers based on Eqs. (6a) and (7a), can be taken as

$$u = U_c[\hat{u}_+(\eta) + \hat{u}_*(\zeta)] + u_\tau[u_+(\eta) + u_*(\zeta)] + \dots \tag{9}$$

where $\mathcal{O}(U_c) > \mathcal{O}(u_\tau)$. As the velocity in the inner layer is of the order of $u_\tau$, without loss of generality, it may be taken that $\hat{u}_+(\eta) = 0$ and Eq. (9) reduces to

$$u = U_c\hat{u}_*(\zeta) + u_\tau[u_+(\eta) + u_*(\zeta)] + \dots \tag{10}$$

Likewise, the composite expansion for the Reynolds stress is

$$\tau = \tau_w[\tau_+(\eta) + \epsilon(R_\tau)\tau_*(\zeta)], \qquad \zeta = y/\Delta(R_\tau) \tag{11}$$

where $\Delta$ is the intermediate length scale and $\epsilon$ a gage function.

The first term in the velocity expansion equation (10) is directly related to the work of Afzal, described in Sec. III, where the velocity in the intermediate layer is of the order of unity. This first term is not mentioned in Ref. 1 as Long and Chen had postulated that the order of this velocity was $u_\tau$. Long and Chen's mesolayer theory ($U_c = 0$) leads to a law of the wall, the intercept of which depends on $R_\tau$, as does $\ln R_\tau$ when $R_\tau$ approaches infinity (see Ref. 2), whereas classical theory and the existing measurements show a constant (universal) intercept.[8] A further justification for $U_c \neq 0$ is described later in this section, where a particular choice of $U_c$ leads to the classical law of the wall.

Substituting Eqs. (10) and (11) into Eq. (1) results in

$$\frac{du_+}{d\eta} + \tau_+ = 1 \tag{12}$$

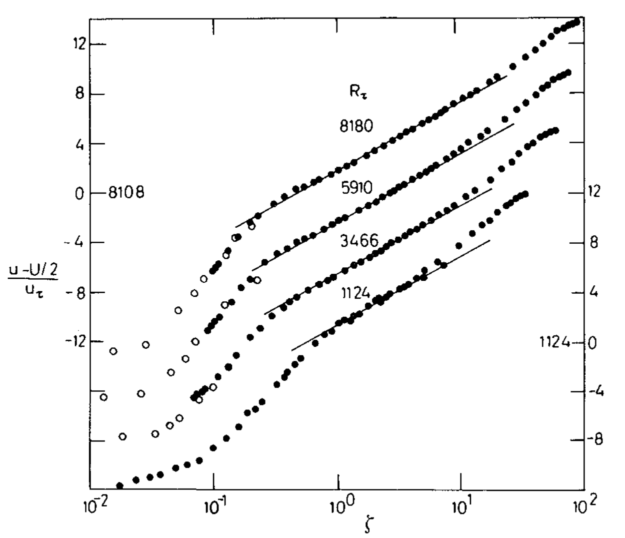


**Fig. 1 Velocity distribution in terms of half-defect law [Eq. (26)] coordinates from data of Ref. 9 (note displaced ordinates).**

$$\frac{d\hat{u}_*}{d\zeta} + \left(\frac{u_\tau}{U_c}\right)\left[\frac{du_*}{d\zeta} - \left(\frac{\Delta}{h}\right)\epsilon R_\tau \tau_* - \left(\frac{\Delta}{h}\right)^2 R_\tau \zeta\right] = 0 \tag{13}$$

In the square brackets, all terms are of the same order provided that

$$\Delta/h = \epsilon = R_\tau^{-1/2} \tag{14}$$

are the intermediate layer variables.[1,2] As $R_\tau \to \infty$ and $u_\tau/U_c \ll 1$, Eq. (13) leads to

$$\frac{d\hat{u}_*}{d\zeta} = 0 \tag{15a}$$

$$\frac{du_*}{d\zeta} + \tau_* - \zeta = 0 \tag{15b}$$

Without loss of generality, the solution of Eq. (15a) can be taken as $\hat{u}_*(\zeta) = 1$.

Matching outer expansion of Eq. (8a) as $Y \to 0$ with composite expansion of Eq. (10) as $\zeta, \eta \to \infty$ for velocity requires

$$U + u_\tau F(Y \to 0) \sim U_c + u_\tau[u_+(\eta \to \infty) + u_*(\zeta \to \infty)] \tag{16}$$

This is a functional equation, the solution of which is[1,7]

$$u = U + u_\tau[(S_1 + S_2)\ln Y - D] \tag{17}$$

$$u = U_c + u_\tau(S_1 \ln\eta + C_1 + S_2\ln\zeta + C_2) \tag{18}$$

$$(U - U_c)/u_\tau = (S_1 + \tfrac{1}{2}S_2)\ln R_\tau + C_1 + C_2 + D \tag{19}$$

where $C_1$, $C_2$, $D$, $S_1$, and $S_2$ are constants. Equation (17) is the classical velocity defect law if

$$S_1 + S_2 = 1/k \tag{20}$$

where $k$ is von Kármán's universal constant. Expressing Eq. (18) in terms of inner variable $\eta$ and using Eq. (19) to eliminate $\ln R_\tau$ we get

$$u = [2(S_1 + S_2)U_c - S_2 U]/(2S_1 + S_2) + u_\tau\{k^{-1}\ln\eta + B$$
$$+ [2(C_1 + C_2)(S_1 + S_2) + DS_2 - B(2S_1 + S_2)]/(2S_1 + S_2)\} \tag{21}$$

the expression for law of the wall. In Eq. (21), a universal constant $B$ is introduced for later convenience. If $U_c = 0$, as in the work of Long and Chen, then in Eq. (21) the reference velocity $-S_2U/(2S_1 + S_2)$ is finite, whereas according to the classical law of the wall the reference velocity should be zero.[8] Therefore, the mesolayer theory[1] does not lead to the classical law of the wall. Equation (21) yields the classical law of the wall

$$u = u_\tau(k^{-1}\ln\eta + B) \tag{22}$$

when $U_c$ in Eq. (21) is chosen as

$$U_c(1 + S_1/S_2) = U/2 + u_\tau[(C_1 + C_2)(1 + S_1/S_2)$$
$$- BS_1/S_2 - (B - D)/2] \tag{23}$$

Based on Eq. (23), Eq. (19) reduces to

$$U/u_\tau = k^{-1}\ln R_\tau + B + D \tag{24}$$

which is the classical skin friction law. Further, the expressions for Reynolds stress obtained from Eqs. (17), (22), and (1) are

$$\tau/\tau_w = 1 - Y \tag{25a}$$

$$\tau/\tau_w = 1 - (k\eta)^{-1} \tag{25b}$$

Expressing Eq. (18) in terms of $\zeta$ and eliminating $\ell n R_\tau$ and $U_c$, we get

$$u = \tfrac{1}{2} U + u_\tau [k^{-1} \ell n \zeta + \tfrac{1}{2}(B-D)] \tag{26}$$

which is an expression for velocity in the intermediate layer. This may be termed as a half-defect law. The expression for Reynolds stress in the intermediate layer obtained from Eqs. (1) and (26) is

$$\tau/\tau_w = 1 - R_\tau^{-\frac{1}{2}} [\zeta + (k\zeta)^{-1}] \tag{27}$$

It is seen that Reynolds stress in Eq. (27) is associated with a maximum, whose magnitude $\tau_{max}$ and location $\zeta_{max}$ are given by

$$\tau_{max}/\tau_w = 1 - 2(kR_\tau)^{-\frac{1}{2}} \tag{28a}$$

$$\zeta_{max} = k^{-\frac{1}{2}} \tag{28b}$$

## V. Results and Discussion

The present three-layer analysis leads to the classical law of the wall, the velocity defect law, and the skin friction law. In the intermediate layer, the reference velocity is half of the velocity at the axis of the pipe/channel and the velocity distribution is governed by a half-defect law. From Eq. (26) it may be seen that $u \to U/2$, $\zeta \to \zeta_{\frac{1}{2}}$ where $\zeta_{\frac{1}{2}}$ is given by

$$\zeta_{\frac{1}{2}} = \exp[-\tfrac{1}{2}k(B-D)] \tag{29}$$

Taking $k = 0.41$, $B \simeq 5$, and $D \simeq 0.8$, Eq. (29) gives $\zeta_{\frac{1}{2}} = 0.42$ whereas the analysis[2] of data gives $\zeta_{\frac{1}{2}} = 0.44$. The data of Coantic[9] in terms of half-defect coordinates of Eq. (26) displayed in Fig. 1 show that substantial logarithmic regions with universal slope and intercept do in fact exist and that Eq. (26) can be represented by

$$(u - U/2)/u_\tau = 5.6 \log \zeta + 1.8 \tag{30}$$

A comparison of Reynolds stress expressions in Eqs. (25) and (27) with the data of Laufer is given in Fig. 4 of Ref. 2. The maxima in Reynolds stress predicted by Eq. (28b) at $\zeta_{max} = 1.56$ is in better agreement with data[1,2] (1.85) when compared with $\zeta_{max} = 1.54$ given by the mesolayer theory.[1] The difference in the theory and measurements can be caused by: 1) the maxima lies outside the logarithmic region in the intermediate layer; 2) the uncertainties in the location of maxima in the measured profile; and 3) uncertainties in measuring small distances, especially very close to the wall.

A comparison of the three-layer theory with the two-layer classical theory shows that the addition of the classical defect law [Eq. (17)] with classical law of the wall [Eq. (22)] leads to the half-defect law [Eq. (26)] for the intermediate layer. However, the similar addition of Reynolds stress expressions [Eqs. (25)] in the inner and outer layers produces an expression where terms of order $R_\tau^{-\frac{1}{2}}$ are different from the Reynolds stress equation (27) in the intermediate layer. This means that for the lowest-order results the two-layer classical theory should suffice as the intermediate layer forms the matching region between the classical two layers.[3,10] Therefore, there is no need to treat the intermediate layer through a separate expansion. However, for the first-order results, (i.e., $R_\tau^{-\frac{1}{2}}$), the intermediate layer is a distinct layer associated with the maximum value of Reynolds stress and it is necessary to consider the three-layer structure of the flow.[10]


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# An Enhanced Flow Visualization Technique for Planar Free Shear Layers


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SCHLIEREN flow visualization has played an important role in the study of organized vortical motions in free shear layers.[1,2] This Note deals with a schlieren technique that enables one to visualize and identify specifically those fluid elements making up the shear layers in a plane jet. The enhanced visualization is achieved with the method by heating the reservoir gas (air) to a particular temperature according to the jet exit Mach number.

A rectangular nozzle of aspect ratio 16.7 (50 mm long, 3 mm wide) was used in the study. For the conditions of interest, hot-wire measurements showed that the flow was very nearly two-dimensional at the jet exit.[3] The flow facility allowed steady conditions to be realized for an hour or longer. Provisions were made to vary the reservoir air temperature by as much as 30°C above the ambient. A conventional schlieren system has been used with a spark light source. All schlieren pictures were taken with spark durations of 2.5 $\mu$s.

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